

A. Bondal

j/w A. Bondal

$$X = \mathbb{P}^1 / \underline{k}$$

$$\text{char } k = 0 \quad (k = \mathbb{C})$$

$$D(\mathbb{P}^1) \simeq D(\cdot \rightrightarrows \cdot)$$

$$A \longmapsto \text{RHom}(E, A)$$

$$E = \mathcal{O} \oplus \mathcal{O}(1) \quad \langle E \rangle = D(\mathbb{P}^1)$$

$$\text{Ext}^{>0}(E, E) = 0$$

$$A = \text{End}(E)$$

$$D(A) \xrightarrow{\sim} D(\mathcal{P})$$

A, \mathcal{P} ab. cats.

2 dif. t-structures in the same cat.

D triangulated

$$\text{Def: } D \supset (D^{\leq 0}, D^{\geq 1})$$

$$1. \text{Hom}(D^{\leq 0}, D^{\geq 1}) = 0$$

$$2. D^{\leq 0}[n] \subset D^{\leq 0} \quad n \geq 0$$

$$3. D^{\geq 1}[n] \subset D^{\geq 1} \quad n \leq 0$$

$$4. \forall A \in D \exists \begin{array}{c} A_{\leq 0} \rightarrow A \rightarrow A_{\geq 1} \\ \uparrow \qquad \qquad \uparrow \\ D^{\leq 0} \qquad \qquad D^{\geq 1} \end{array} \text{ triangle}$$

$$\Rightarrow D^{\geq 1} = (D^{\leq 0})^\perp$$

$$\mathcal{C} = D^{\leq 0} \cap D^{\geq 1}[1] \text{ heart is ab. cat.}$$

Example: $D = D(A)$

$$D^{\leq 0} = D^{\leq 0}(A), \quad D^{\geq 1} = D^{\geq 1}(A) \Rightarrow \mathcal{C} = A$$

$$f: X \rightarrow Y \quad X, Y \text{ alg. var.}$$

$$\bullet Rf_* \mathcal{O}_X = \mathcal{O}_Y$$

$$\bullet \dim f \leq 1$$

$\bullet f$ proper

$$D(X)$$

\cup

$$\mathcal{C}_f = \langle A \in D(X) \mid Rf_* A = 0 \rangle$$

null cat.

$$(\mathcal{C}_f^{\leq 0}, \mathcal{C}_f^{\geq 1})$$

restriction of the standard t-structure

(BBD):

$$A \xrightarrow{i_*} D \text{ admissible } (\exists i^!, i^*)$$

$$\begin{array}{ccc} A & \xrightarrow{i_*} & D & \xrightarrow{q} & B \\ \downarrow i^* & & \downarrow i^* & & \downarrow i^* \\ U & & U & & U \end{array}$$

$(A^{\leq 0}, A^{\geq 1}) \quad (B^{\leq 0}, B^{\geq 1})$

$\exists!$ $(D^{\leq 0}, D^{\geq 0})$ s.t. i_* and q are t-exact

In the case above:

$$C_f \rightarrow D(X) \rightarrow D(Y) \quad \text{stand. t-str.}$$

restrict.
t-str.

You get a t-structure in $D(X)$ which is not the standard one

$$D(X) := D_{\text{coh}}^b(X)$$

$$p \in \mathbb{Z}$$

~~...~~

$${}^p D^{\leq 0} = \{ A \in D(X) \mid Rf_* (A) \subset D(Y)^{\leq 0}, \text{Hom}(A, E) = 0 \forall E \in \mathcal{E}_f^p \}$$

(In our case above $f: P^1 \rightarrow *$)

$$p = -1, 0$$

$${}^p \text{Coh}(X)$$

under t.f. conditions:

- $Y \neq \text{Spec } R$ (*)
- or
- f proj.
- X, Y quasi proj.
- Ex: ---

Consider case (*):

$\exists M \in {}^{-1} \text{Coh}(X)$ M a proj. generator for ${}^{-1} \text{Coh}(X)$ (vect. bundle on X)

$$N = M^* \quad \text{"} \quad \text{"} \quad \text{for } {}^0 \text{Coh}(X)$$

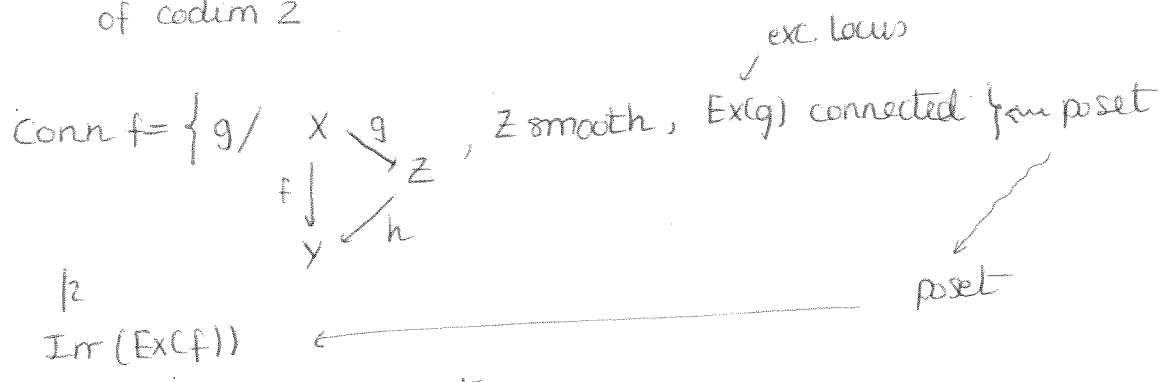
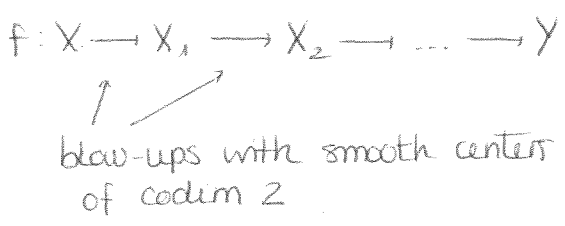
$$\Rightarrow {}^{-1,0} \text{Coh}(X) \simeq A_P\text{-mod} \quad A_P = \text{End } P \quad P = M, N$$

Goal: Find a t-structure with a canonical local injective generator

$X \xrightarrow{f} Y$ ~~with the same conditions as above~~

- f proj.
- f birational
- X, Y smooth
- $\dim f \leq 1$

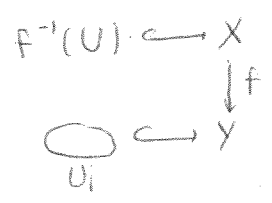
Thm: (Danilov)



Def: Relative tilting ^{object} w.r.t. f $A \in D(X)$ s.t.

- $Rf_* R\text{End}(A) \in \text{Coh}(Y)$
- $\exists U_i \subset Y$ open $\cup U_i = Y$

$$D(f^{-1}(U_i)) \simeq \langle A |_{f^{-1}(U_i)} \rangle_{\oplus}$$



$A_A \simeq_{\text{qis.}} Rf_* R\text{End}(A)$
 \uparrow sheaf of alg. $/ Y$

Thm: $D(X) \simeq D(A_A\text{-mod})$

Thm: under the conditions above:

$A \in D_X$ is an f -relative tilting generator

$$A = \omega_f \oplus \left(\bigoplus_{g \in \text{Conn} f} \omega_g |_{D_g} \right)$$

