

Calaque

Derived symplectic Poisson geom

Motivation A reductive Σ oriented topological surface.

$Loc_G(\Sigma)^{sm} \leftarrow$ smth part of moduli of G -local system

\hat{L} has symplectic structure. Why?

$$T_{\text{tot}} Loc \cong H^1(\Sigma, \mathfrak{g})$$

Pairing: Use Cartan-Killing form (any non-deg Σ -form)

$$H^1(\Sigma, \mathfrak{g})^{\otimes 2} \longrightarrow H^2(\Sigma, \mathbb{C}) \cong \mathbb{C}.$$

Closedness is hard.

If $\Sigma = \partial M$, M 3-manifold, have

$$\text{"} Loc_G(M) \subset Loc_G(\Sigma)^{sm} \text{"}$$

really, all the loc sys on Σ that extend to M . This is Lagr.

Likewise, replace $\Sigma \leftrightarrow K3$ surface

$$Loc_G \leftrightarrow Bun_G$$

Then $Bun_G(K3)^{sm}$ symplectic, (Mukai)

If $K3 \hookrightarrow M$ Fano 3-fold, $[K3] = [w^2]$

have " $Bun_G(M) \subset Bun_G(K3)^{sm}$ " is Lagrangian. (Tyurin)

Really, need cohom. constants on H^2 of G -bundles you consider

One goal: Set up a general framework for such results

- get rid of " " quotation marks
(includes saying something abt non-smth locus)
- quantize moduli spaces

If $[P]$ is non-smth part of $Bun_G(K3)$, then

$$T_{[P]} Bun_G(K3) \cong H^0(K3, ad P) \oplus H^1(K3, ad P)$$

\sim
 isotropy/
 autom gp
 of \mathcal{P}
◦
orham decp

so need something to pair against the deg-1 "H⁰" part — incorporate H² in deg 1.

This involves derived enhancement

$$Bun_G(K3)$$

st

$$T_{[P]} Bun_G(K3) \cong H^{*+1}(K3, ad P)$$

That's the point of derived geometry.

Diff geom on edges: $A \in \text{edges}_{\leq 0}$ rings have negatively graded bits, differential.
 By computation for stacks built from edges.
 (Affines.)

Assume A coherent-invert, quasi-free, so

$$A \cong (\text{Sym } V^*, d). \quad (\text{Everything char } 0)$$

Deligne "algebra":

$$DR(A) := \text{Sym}_A(\Omega_A^1[-1]) \quad \text{a graded mixed complex.}$$

- auxiliary grading called "weight"

$$\text{i.e., } \text{Sym}^d, \quad d = \text{weight.}$$

- another differential coming w/ d_A

$$E = \text{universal derivation, } d_{DR}$$

of degree +1 cohomology.

~~degree~~ +1 weight.

$$\text{i.e., } \text{deg } 1, \text{ wt } 1.$$

- $d_A =: d_{\text{int}}$ has $\text{deg } 0, \text{ wt } 0$.

Gr Mix Complex = $C_{p \times}^{E-gr}$ has model structure, just transferred from $C_{p \times}^{gr}$.

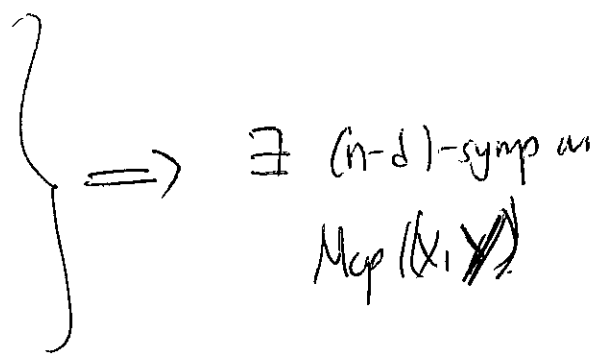
$$C_{p \times}^{E-gr} \xrightarrow{\text{forget } E} C_{p \times}^{gr} \quad \underbrace{\hspace{10em}}_{\text{induces model structure.}}$$

Recipe for new symp stacks. Since $\text{symp} \Rightarrow \text{Pois}$, get quantization.

X

\mathcal{O} -compact derived stack.
d-oriented

n-symp stack



\exists (n-d)-symp as $\text{Map}(X, Y)$

we $\text{Map}(X, Y)$ Artin, loc fin pres stack

[in cat of derived stacks, even if X, Y not derived.

in ex. of \mathcal{O} -compact stacks: • X smth proj d-CY

• $X = \sum_{B_i \in \text{Betti}}$ ~~loc syst~~ \sum d-dim oriented top mtd.

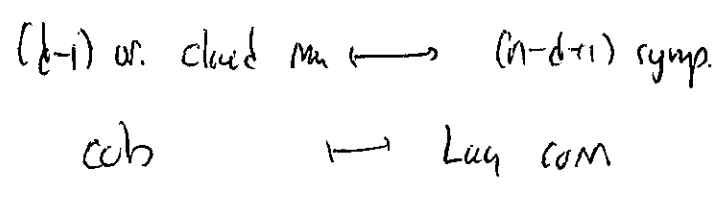
we $Y = B\mathbb{G}_a \Rightarrow \text{Map}(X, B\mathbb{G}_a) = \text{Bun}_G(X)$ (2-d)-symp

ex: d=2, X 2-CY, ex $\mathbb{C}P^1$, the $\text{Bun}_G(X)$ is 0-symp, so symp as smth part.

and $\text{Map}(\sum_{B_i} B\mathbb{G}_a) = \text{Loc}_G(\Sigma)$.

Compatible w/ standards:

Y n -symp. stack. The $\text{Map}(\tau)_{B, Y}$ is a TFT at level in
Lagrangian cobordisms
 Lag_{n-d+1}



Quantize Y , and then the TFT.

Strategy for $\text{Loc}_G(\Sigma)$: 1) Quantize Bh $\text{Rep}(G)$ $(n\text{-symp } Y$
 \parallel $\text{Map}(\Sigma_B, Bh)$ so \downarrow $\text{1st orb det of } \text{Sh}(Y)$
 quantum group is $\text{as } \mathbb{F}_1\text{-cat}$
 $\mathbb{F}_2\text{-det ; } \text{Rep}(U_q \mathfrak{g}).$

2) Quantize $\text{Map}(\tau, Bh)$ TFT, via factorization theory.

One case worked out: Ben-Zvi, Bruchter, Jardine. Compact

$$\int_{\Sigma} \text{Rep}(U_q \mathfrak{g})$$

- and prove:
- gives back quantization, in above case, of charac varieties.
 - \int_{Σ} are $\text{Rep}(\text{some alg})$, the algebra is alg of sheaves on combinatorial quantization of Chern-Simons Alg. (Alekseev)