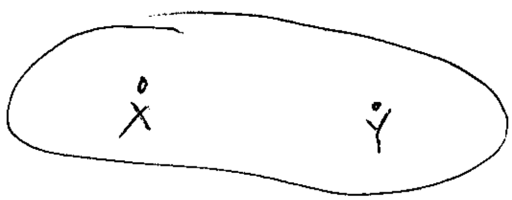
Segal

Pfaffian-Grassmannian correspondence

Two CY 3-folds X, Y

Same Hodge #'s, so have same mirrors (guess).

Writing Picard-Fuchs eqns, they were equal! (closed string MS) 98



← single component

PF family

So X, Y connected by some variation of Kähler str.

ie, $D^b Coh(X) \simeq D^b Coh(Y)$ (B branes equal)

turns out to be true!

Thm (Borisov, Caldararu, Kuznetsov) 2006

But X, Y not birational!

$X \xleftrightarrow{\text{flop}} Y \overset{?}{\Longrightarrow} D^b$ equivalent

But not birational, so can't be flopped.

Physics breakthrough shows more:

Hori-Tong: Wrote family of QFTs from Σ model on X
a "Kähler"
to " " " Y.

gauged linear Σ model, non-abelian gauge gp.
Reinterpreted mathem. by Addington-Donovan-S., new proof of D^b equivalence

Hori again: $\exists$ dualities btw non-ab gauged linear Σ models
explaining this · Pfaff-Grass correspondence
, and others
[instances of homological projective duality (HPD)

Pfaffian varieties + HPD

Fix V vec space $\quad \Lambda^2 V^* \supset \{\text{forms of rank} \leq 2q\} = \widetilde{\mathrm{Pf}_q}$

$\Lambda^2 V^*$ " 2-forms; fixed upper rank; $\widetilde{\mathrm{Pf}_q}$ a cone

$\rightsquigarrow$ projectivization $\mathrm{Pf}_q \subset \mathbb{P}(\Lambda^2 V^*)$

Very singular, when ranks drop, NOT complete intersections.

Projective dual is another Pfaffian: $\mathrm{Pf}_s \subset \mathbb{P}(\Lambda^2 V)$, and

$$2s+2q = \begin{cases} \dim V & (\dim V \text{ even}) \\ \dim V - 1 & (V \text{ odd}) \end{cases}$$

HPD summary:

Fix linear slice

$$L \subset \Lambda^2 V^* \quad \text{subspace}$$

$$\Rightarrow L^\perp \subset \Lambda^2 V$$

Cut variety w/ slices:

$$\mathrm{Pf}_q \cap \mathbb{P}L \qquad \mathrm{Pf}_s \cap \mathbb{P}L^\perp$$

and consider

$$D^b(\mathrm{Pf}_q \cap \mathbb{P}L) \qquad D^b(\mathrm{Pf}_s \cap \mathbb{P}L^\perp)$$

These should be approx. equal: $\simeq$

Really, one embeds to other as admiss subcat, and complement is "boring" — Lefschetz hyperplane style.

(Really: contain common interesting piece.)

Pf_q singular, so replace w/ non-comm resolutions

NCCR - noncomm Crepant resolns
(sheaves of algebras on variety)

Proved for $q=1$, $\dim V \leq 7$

- Note $Pf_2 = Gr(2,V)$.
- Case $\dim V = 7$ is the X, Y from beginning.

<u>Hori duality</u>: GLSM = Lie gp G acting on vec space.

Given Q symp vec sp $\dim Q = 2q$,

$$\mathrm{Hom}(V,Q) \big/ Sp(Q) \twoheadrightarrow Pf_q \subset \Lambda^2 V^*$$

and $\left[\mathrm{Hom}(V,Q) / Sp(Q) \right]$ is resoln of Pf_q

Dual model: S symp vec sp, $2q + 2s = \dim V - 1$

$Sp(S) \circlearrowright \mathrm{Hom}(S,V) \times \Lambda^2 V$ w/ $Sp(S)$-inv fcn W.

$\mathrm{hom}(S,W) / Sp(S) = Pf_S \subset \Lambda^2 V$

B brane category on twisted(?) manifolds

$$X - \text{model} \Rightarrow \overset{\text{B branes.}}{\cancel{\text{K}} \text{ stack } D^b(X)}$$

but $D^b\left(\text{Hom}(V,Q)/Sp(Q)\right)$ too big.

Recall $Sp(Q)$ reps $\leftrightarrow$ Young diag of ht $\leq q = \frac{1}{2}\dim Q$

$$\mathrm{Br}\left(\left[\mathrm{Hom}(V,Q)/Sp(Q)\right]\right) \overset{\substack{\dim V \text{ odd} \\ \text{Thm}}}{\simeq} \mathrm{Br}\left(\left[\mathrm{Hom}(S,V)/Sp(S)\right] \times \Lambda^2 V^\vee, W\right)$$

$$\| $$

$$\langle \mathbb{S}^{\langle Y \rangle} Q,\ y \leq S \rangle$$

looks like Koszul duality of cned(?) on right.

a vec bdl
supp. on Young diag Y,
on $\left[\mathrm{Hom}(V,Q)/Sp(Q)\right]$

<u>rmk</u> give $\Lambda^2 V^\vee$
R-charge 2.

This gives W R-charge 2,
hence $\mathbb{Z}$-graded.