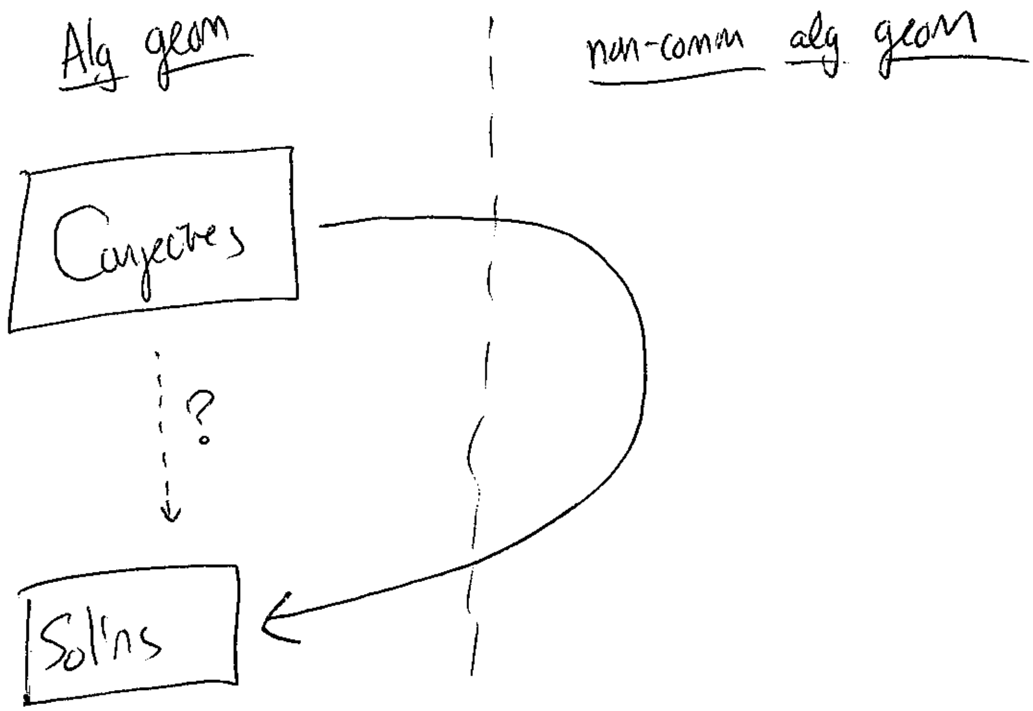
Tabuada



K base field, char > 0

X smth proj K-sch

H^* Weil cohom, Künneth projector $\pi_X^i : H^*(X) \to H^i(X) \to H^*(X)$

$h(-)_{\mathbb{Q}} : SmProj(K)^{op} \longrightarrow Chow(K)_{\mathbb{Q}}$

Chow motives category

<u>Conj $C^+(X)$</u> (Groth deck)

The even projector $\pi_X^+ := \sum_i \pi_X^{2i}$ is <u>algebraic</u>

<u>Conj $V(X)$</u> (Voevodsky '95).

$Z^*(X)_{\mathbb{Q}}/_{\text{smash nilp. equiv}} = Z^*(X)_{\mathbb{Q}}/_{\text{numer. equiv}}$

<u>Conj $D(X)$</u> (Groth)

$Z^*(X)_{\mathbb{Q}}/_{\sim \text{hom}} = Z^*(X)_{\mathbb{Q}}/_{\sim \text{num. equiv.}}$

<u>Conj $S(X)$</u> (Kimura) $h(X)_{\mathbb{Q}}$ is Schur finite

Rmk

- $\sim_{nil} \supset \sim_{hom} \supset \sim_{num}$.

So $V(X) \Rightarrow D(X)$

- $D(X \times X) \Rightarrow C^+(X)$

- C^+, D, V, S known when $\dim X \leq$ 2, 4, 2, 1.

Non comm alg geom

dg cat $A_{/k}$ smth $\Leftrightarrow$ $A \in D_c(A^{op} \otimes A)$

prop $\Leftrightarrow$ $\sum_i \dim(H^i A(*,*)) < \infty$ $\forall *$.

Rmk X smth prop $\Leftrightarrow$ $\mathrm{Perf}_{dg}(X)$ smth prop.

Defn $\mathrm{dgCat}_{sp}(k) =$ smth prop dg Cat$/k$.

NC conjectures:

A smth prop

$HP^{\pm} \rightsquigarrow \pi_A^{+} : HP^{\pm}(A) \to HP^{+}(A) \to HP^{\pm}(A)$

periodic reg cyclic homology   projector

Rmk char 0, HKR

$\Downarrow$

$HP^{+}(\mathrm{Perf}(X)) = \bigoplus_{\text{even}} H^{\bullet}_{dR}(X)$

$HP^{-}(\mathrm{Perf}) = \bigoplus_{\text{odd}} H^{\bullet}_{dR}(X)$

$U(-)_{\mathbb{Q}} : \mathrm{dgcat}_{sp}(K) \longrightarrow \mathrm{NChow}(K)_{\mathbb{Q}}$

noncomm Chow motives,
hom K_0 gps $/\mathbb{Q}$,
compos = $\otimes$ bimod

Conj $\cdot$ $C_{NC}(A)$ π_A^{+} alg

$K_0(A) \times K_0(A) \longrightarrow \mathbb{Z}$

$(M, N) \longmapsto \sum \mathrm{Ext}^i(M, N)$

smth prpr
$\Downarrow$ Bondal-Kapranov
Serre func
$\Downarrow$

$\cdot$ D_{NC} $\quad K_0(A)_{\mathbb{Q}}/\sim_{\text{homology}} \cong K_0(A)_{\mathbb{Q}}/\sim_{\text{numerical}}$

$\cdot$ V_{NC} $\quad '' /\sim_{\text{nilp}} \cong '' /\sim_{\text{numerical}}$

$\cdot$ S_{NC} $\quad V(A)_{\mathbb{Q}}$ is Schur finite

Thm X smth proj K-sch.

X cojecture $\Longleftrightarrow$ $A = \mathrm{Perf}_{dg}(X)$ cojecture

for $C^{\dagger}, D, S, V$.

We focus on V.

Homol. Projective Duality (HPD):

Fix $f : X \to \mathbb{P}(V)$, $\mathcal{O}(1) := f^* \mathcal{O}_{\mathbb{P}(V)}(1)$

Assume $\mathrm{Perf}(X) = \langle \mathcal{A}_0, \mathcal{A}_1(1), \ldots, \mathcal{A}_{i-1}(i-1) \rangle$

Lefschetz decomp. ie,

$$\mathcal{A}_0 \supseteq \cdots \supseteq \mathcal{A}_{i-1}$$

and semiorthogonal decomp.

Incidence quadric: $Q \subset \mathbb{P}(V) \times \mathbb{P}(V^*)$

univ hyperplane section: $\mathcal{H} := X \times_{\mathbb{P}(V)} Q \subset X \times \mathbb{P}(V^*)$

$$\mathrm{Perf}(\mathcal{H}) \overset{\text{thm}}{\simeq} \langle \underset{\substack{\| \\ \text{HPD dual cat of } X}}{\mathcal{C}}, \mathcal{A}(1) \boxtimes \mathrm{Perf}(\mathbb{P}(V^*)), \ldots, \mathcal{A}_{i-1}(i-1) \boxtimes \underset{\mathbb{P}(V^*)}{\mathrm{Perf}} \rangle$$

In fact, $\mathcal{C} \simeq \mathrm{Perf}(Y)$ gives $Y \longrightarrow \mathbb{P}(V^*)$.

or $\simeq \mathrm{Perf}(Y, \mathcal{F})$

Y called HPD dual of X.

$$L \subset V^* \rightsquigarrow L^{\perp} := \mathrm{Ker}(V \twoheadrightarrow L^*).$$

$$\rightsquigarrow X_L := X \times_{\mathbb{P}(V)} \mathbb{P}(L^{\perp})$$

$$Y_L := Y \times_{\mathbb{P}(V^*)} \mathbb{P}(L)$$

<u>Thm</u> (HPD invariance)

Assume (i) X_L, Y_L smth

(ii) $\mathrm{codim}(X_L) = \dim(L)$

$\mathrm{codim}(Y_L) = \dim(L^{\perp})$

(iii) $V_{NC}(\mathcal{A}_{0,dg})$ holds.

Then $V(X_L) \Longleftrightarrow V(Y_L)$.