~~$R = k[x,y]/(xy)$~~

~~$N^{-1}R = R[z^{-1}, y^{-1}]$~~

Wemyss

Flop like a mutation:

Take out a curve, put back at most one.

(Like mutation.)

$$X \xrightarrow{f} X_{con} \xleftarrow{f^+} X^+$$

contract curve C,

$f|_{X \setminus C}$ is $\cong$.

f^+ a flop of f iff

$\forall\ \mathcal{L} = \mathcal{O}_X(D)$ on X

st $-D$ is f-nef,

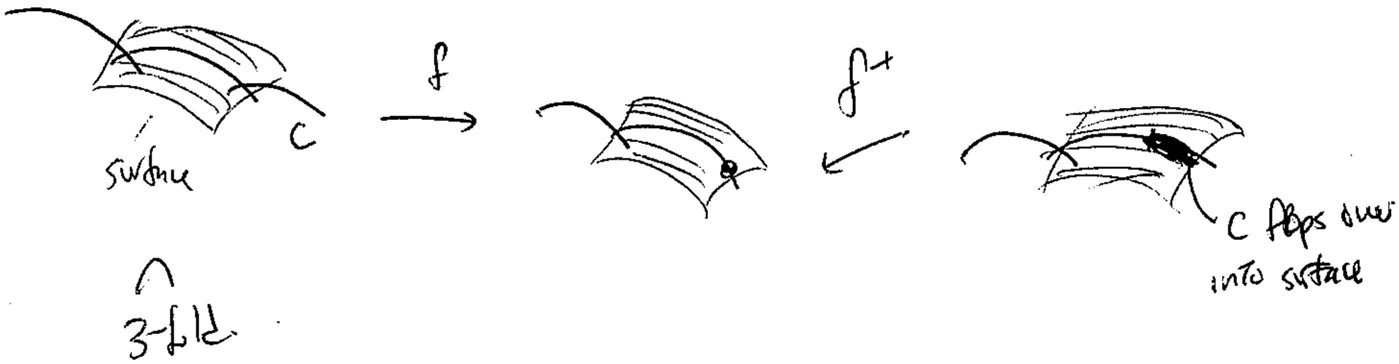
the transform of D is Cartier,

f^+-nef.

using ∩ thy

to make sure you're

getting right answer.



Hard to iterate flops. $\rightsquigarrow$ homological MMP via mutations.

For 3-folds (unlike 2-folds)

- fibre need not be ADE
- two curves $\neq A_2$
- objects no longer spherical.
  Need to deform; $\mathcal{O}_i$ like very technical.

Elephant conjecture:

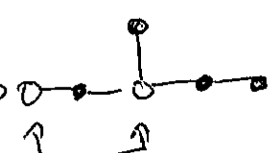
$$\begin{tikzcd} \cdot \arrow[d] \arrow[r, "\text{p.b.}"] & u \subset X \arrow[d, "\text{flop}"] \\ \text{surface} \arrow[r] & \text{3 fold} \end{tikzcd}$$

R/g $\quad$ $\operatorname{Spec} R$

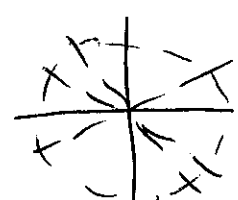
dim 2:
$\mathbb{C}^2/G$, $G \subset GL_2(\mathbb{C})$
above 0, have
$\bigcup \mathbb{P}^1 = C = \text{fibre}^{\text{red}}$,
ADE sing. Braid gp action by T_{E_i}
$E_i = \mathcal{O}_{\mathbb{P}^1}$

p.b. is some surface sing. $\rightsquigarrow$ shaded Dynkin diag, by where nodes come from.

3-fold $\cap$ surface loses info, but may still be useful.

Ex E_6  , $\Rightarrow$ 2 roots in root system $\Rightarrow$ plane w/ reflecting hyperplanes

2 shade pts



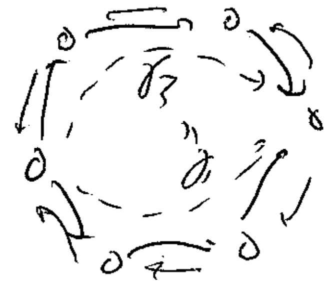
10 chambers in $\mathbb{R}^2$

Ex  $\to$ 32 chambers in $\mathbb{R}^3$.

Deligne Gpd: • dot in every chamber

• edge ∀ adjacent chambers/walls

• relation: two min. paths identified.



Complexified complement has π_1 = vertex gp of grpoid.

$\|$

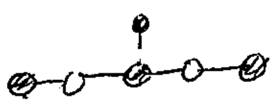
$\mathbb{G}$

<u>Thm</u> $X \longrightarrow X_{con}$ flop, each curve individually floppable

(1) Gp homom $\rho : \pi_1(\mathbb{G}) \longrightarrow \operatorname{Aut} D^b \operatorname{Coh} X$

(2) ρ injective

Unlike Seidel-Thomas, no gen/rel on $\pi_1(\mathbb{G})$, no formula for flop functors.

Make affine gpd $\mathbb{G}_{aff}$ for tiling; given by  shaded diag.

<u>Thm</u> $\rho : \pi_1(\mathbb{G}_{aff}) \longrightarrow \operatorname{Aut} D^b \operatorname{Coh}(X)$

Mirror unknown for curves that deform - no longer toric.