Neeman

$\langle G \rangle_n$

$\langle G \rangle_{n+1} = \langle G \rangle_1 * \langle G \rangle_n$

- Definition 1: (i) G is a classical generator if $\mathcal{T} = \bigcup_{n=1}^{\infty} \langle G \rangle_n$

  (ii) G is a strong generator if $\exists\, n$ with $\mathcal{T} = \langle G \rangle_n$

- Definition 2: $\mathcal{T}$ is regular if $\exists$ a strong generator $G \in \mathcal{T}$
- Definition 3: Let R be a noeth. $\overset{\text{comm.}}{\wedge}$ ring, and $\mathcal{T}$ an R-linear triangulated cat. Then $\mathcal{T}$ is proper$/_R$ if $\bigoplus_i \mathrm{Hom}(X, Y[i])$ is a finite R-module $\forall\, X, Y \in \mathcal{T}$.
- Theorem 1 (B-VdB):

  Suppose $\mathcal{T}$ is regular and proper over R. Then a functor $\overset{R\text{-linear}}{F : \mathcal{T} \longrightarrow R\text{-Mod}}$ is representable iff it is homological and $\bigoplus_{i\in\mathbb{Z}} H^i(X)$ is a finite R-module $\forall\, X$.

- Corollary: If $\mathcal{T}$ is as above and $\mathcal{A} \subset \mathcal{T}$ is a full triangulated subcat. and the inclusion $\mathcal{A} \longrightarrow \mathcal{T}$ has either a left or a right adjoint, then it has the other adjoint.
- Examples: If k is a field, X is a regular scheme proper over k, then $\mathcal{T} = D^b(\mathrm{Coh}\, X)$.

  $\mathcal{T} = \langle \mathcal{A}, \mathcal{B} \rangle$ SOD

X sep. noeth. scheme

$D^{perf}(X)$	$D^b(\mathrm{Coh}\, X)$
$X = \mathrm{Spec}\, R$: $D^{perf}(X)$ is regular iff R is regular	
If X is smooth over a field k $\longrightarrow$ Regular $\longrightarrow$	(same cat.) Regular
	if X is of finite type over perfect field k then $D^b(\mathrm{Coh}\, X)$ is regular

- Theorem: $D^{perf}(X)$ is regular iff X is regular and finite dimensional
- Theorem: $D^b(\mathrm{Coh}\,X)$ is regular if every closed subset of X has a regular alteration.

$\forall\, a \leq b,\ n \in \mathbb{N},\quad \langle G \rangle_n^{[a,b]} \qquad , \quad \overline{\langle G \rangle}_n^{[a,b]}$

- Theorem: $E \in D_{qcoh}(X)^{\leq 0} \Rightarrow \exists$ triangle $D \to E \to F \to \Sigma D$ with $F \in D_{qch}(X)^{\leq m}$ and $D \in \overline{\langle G \rangle}_n^{[a,b]}$ (integers depend on X)
- Theorem: if $U \to X$ is an open inmersion and E is in $D^{perf}(U)$, then $Rj_* E \in \overline{\langle G \rangle}_n^{[a,b]}$.