SEMI-ORTHOGONAL DECOMPOSITION OF GIT QUOTIENT STACKS

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Aim: SOD of $D(X/G)$ consisting of NCR of "quotient subvarieties" of $X/\!/G$

In this talk:

G reductive group

$X = W^\vee$ W finite dim G-representation ($k[X] = SW$)

In general X smooth variety s.t. the good quotient exists

$D(\Lambda) = D^b_f(\Lambda)$, $D(X/G) = D^b_{coh}(X/G)$

1. NC(crepant) resolutions of quotient singularities

a) G finite group

U fin dim G-rep. $M(U) = (k[X] \otimes U)^G$ module of covariants

$M(\mathrm{End}\, U)$ algebra of covariants

Thm [Auslander]

$U = \bigoplus_{V \in \mathrm{Irr}\, G} V$ $U \otimes k[X]$ proj gen. $\mathrm{mod}(k[X], G)$

- $\mathrm{gldim}\, [M(\mathrm{End}\, U)] < \infty$
- if G does not fix any hyperplane point-wise

$M(\mathrm{End}\, U) \cong \mathrm{End}_{k[X]^G}(M(U))$ NCCR of $k[X]^G$

b) G reductive group

Problem: $\mathrm{mod}(k[X], G)$ does not have a proj. gen.

$\rightsquigarrow$ construct complexes which relate projections

Thm $\exists\, U \neq 0$ fin. dim. G-repr. such that $\mathrm{gldim}\, M(\mathrm{End}\, U) < \infty$

- G acts generically on X if $\mathrm{codim}(X - X^s, X) \geq 2$

$X^s = \{x \in X / Gx \text{ closed } G_x = \{1\}\}$

- W T-representation $(\beta_i)_{i=1}^n$

W is quasi-symmetric if $\forall\, 0 \in \ell \in X(T)_{\mathbb{R}}$ ~ $\sum_{\beta_i \in \ell} \beta_i = 0$

Thm W is generic + quasi-symmetric + "generacy condition" $\Rightarrow$

$\Rightarrow \exists\, U \neq 0$ fin dim G-repres. such that $\mathrm{End}_{k[X]^G}(M(U))$

NCCR of $k[X]^G$